

# Planes

## What is the definition of a plane?

A **plane** is a flat surface that extends in all directions. Can be defined by 3 non-colinear (not on a line) points or by a point and a vector (which is perpendicular to the plane).

## What is the equation for a plane?

The general equation for a plane is:

$$Ax + By + Cz + D = 0$$

An alternate form of this equation is:

$$A'x + B'y + C'z + D' = 0$$

$$A' = A/d, B' = B/d, C' = C/d, D' = D/d$$

$$\text{where } d = \sqrt{A^2 + B^2 + C^2}$$

## How can I find the normal vector to a plane?

Given three points (P1, P2, P3) the normal vector N is given by:

$$N = [P2 - P1] \times [P3 - P1]$$

where the operation "X" is the cross product.

Given the equation for a plane:

$$Ax + By + Cz + D = 0$$

the vector [A B C] is normal to the plane.

## How can I find the equation for a plane?

Given three points: P1, P2, P3

- Find the normal vector,  $N = [A \ B \ C]$ .
- Find the coefficient D, by taking the dot product of N with any point P in the plane and negating the result:

$$N \cdot P = -D$$

- Use the coefficients found (A,B,C,D) in the general equation for a plane.

## How can I find the intersection of two planes?

The intersection of two non-parallel planes is a line. The equation for this line can be found by solving the simultaneous equations of the planes. This can be done using a general method such as Gaussian elimination.

## How can I find the intersection of a plane and a line?

The intersection of a plane and a line that are not parallel is a point. This point can be found by solving the simultaneous equations for the plane and the line using a general method such as Gaussian elimination.

## How can I determine if a given point is in a particular plane?

Given a point  $P = (x, y, z)$ , and a plane with normal  $N$  and point  $P_1$ :

$$N \cdot [P - P_1] = 0$$

if  $P$  is in the plane.

## How can I determine if a given line is in a particular plane?

Given two points on the line,  $P_1$  and  $P_2$ :

If both points are in the plane, the line is in the plane.

Given a line defined by vector  $V$  and point  $P$ :

If  $P$  is in the plane and  $V$  is perpendicular to  $N$  (the normal to the plane), the line is in the plane.

## What is a half-plane?

A line which lies in a plane divides that plane into two **half-planes**. A half-plane expresses an inequality. For example, for the equation  $x + y > 2$ , all the points in the  $x$ - $y$  plane that satisfy the inequality are in one half-plane and those that do not are in the other. The dividing line is the equation  $x + y = 2$ .